NUCLEI

NUCLEAR COMPOSITION

$_{11}Na^{23}$ mass number = p+n=A

Atomic number = Z

Z = proton = 11

Neutron = 23-11=12

01 NUCLEAR RADIUS

$R \propto A^{1/3}$ $R = R_0 A^{1/3}$

$R_0 = 1.2$ fm

02 NUCLEAR VOLUME

$V \propto A$

03 NUCLEAR DENSITY

Independent of A

It is same for all atoms

$\rho = 2.3 \times 10^{17}$ kg/m^3

MASS DEFECT

$P + n \rightarrow$ nucleus

mass of nucleus $< \Sigma(Z)m_p + \Sigma(A-Z)m_n$

$\Delta M = [Zm_p + (A-Z)m_n] - m_{nucleus}$

BINDING ENERGY

$B.E = \Delta Mc^2$

$B.E = \Delta M$ (in amu) $\times 931$ MeV

$\underbrace{A + B}_{m_R} \rightarrow \underbrace{C + D}_{m_P}$ $m_R > m_P$

$\Delta m = m_R - m_P$

$B.E = \Delta m \times 931$ MeV

Q valve = $B.E_P - B.E_R$

BINDING ENERGY PER NUCLEON

$E_{bn} = E_b / A$

As E_{bn} increases the nucleus becomes more stable

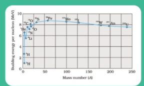
B.E / nucleon given

$\overset{A_1}{A} + \overset{A_2}{B} \rightarrow \overset{A_3}{C} + \overset{A_4}{D}$

B.E / nucleon E_A E_B E_C E_D

Q value = $(E_C A_3 + E_D A_4) - (E_A A_1 + E_B A_2)$

BINDING ENERGY CURVE



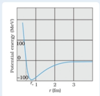
- The curve has a maximum of about 8.75 MeV for A=56
- For nuclei of middle mass number (30<A<170) E_{bn} is a constant
- E_{bn} is lower for both light nuclei (A<30) and heavy nuclei (A>170)

If radius of the $^{27}_{13}Al$ nucleus is taken to be R_{Al}, then the radius of $^{125}_{53}Te$ nucleus is nearly

(a) $\frac{53^{1/3}}{13} R_{Al}$ (b) $\frac{5}{3} R_{Al}$ (b) $\frac{3}{5} R_{Al}$ (a) $\frac{13^{1/3}}{53} R_{Al}$

NUCLEAR FORCE

01 Strongest force existing in nature



02 Nuclear force is short ranged

r>0.8 fm - attractive

r<0.8 fm - repulsive

03 Nuclear force is charge independent

$F_{p-p} = F_{n-n} = F_{p-n}$

Alpha Decay

$^A_ZX \rightarrow ^{A-4}_{Z-2}Y + \alpha$ particle + Q

$Q = [m_x - m_y - m_\alpha]C^2$

Momentum Conservation

$0 = v_y(A-4) - v_\alpha 4$ $v_y = \frac{v_\alpha 4}{A-4}$

K.E

$Q = K.E_\alpha + K.E_y$

$K.E = \frac{P^2}{2m}$; $K.E \propto \frac{1}{m}$ → K.E of α particle is more than daughter nucleus

$K.E_y = \frac{4}{A} \times Q$ $K.E_\alpha = \frac{A-4}{A} \times Q$

No.of alpha decays $n_\alpha = \frac{A-A'}{4}$

Beta Decay

β^- Decay | β^+ Decay

β^- Decay

$n \rightarrow p + \beta^-$

$^A_ZX \rightarrow ^A_{Z+1}Y + \beta^-$

Atomic number increases by one and mass number remains same

β^+ Decay

$p \rightarrow n + \beta^+$

$^A_ZX \rightarrow ^A_{Z-1}Y + \beta^+$

Atomic number decreases by one and mass number remains same

Gamma Decay

No change in atomic number & mass number

Penetrating power : Gamma > Beta > Alpha | Ionizing power : Alpha > Beta > Gamma

In the uranium radioactive series the initial nucleus is $_{92}U^{238}$ and final nucleus is $_{82}Pb^{206}$. When the uranium nucleus decays to lead, the number of a-particles emitted is α and the number of β-particles emitted =

(d) 6, 8 (d) 8, 6 (d) 16, 6 (d) 32, 12

A nucleus of uranium decays at rest into nuclei of thorium and helium. Then:

(a) The helium nucleus has less kinetic energy than the thorium nucleus.

(b) The helium nucleus has more kinetic energy than the thorium nucleus.

(c) The helium nucleus has less momentum than the thorium nucleus.

(d) The helium nucleus has more momentum than the thorium nucleus.

Decay law at radio activity

$\frac{dN}{dt} = \lambda N$

$N_t = N_0 e^{\lambda t}$ (N→No. of undecayed nuclei)

Activity

$A = A_0 e^{-\lambda t}$

$A = -\frac{dN}{dt}$

$A = \lambda N$

Time at which ratio of nuclei will be 1/e

$t = \frac{2.303}{\lambda} \log \frac{N_0}{N}$

$t_{1/2} = \frac{0.693}{\lambda}$

Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time:

(a) $\frac{1}{9\lambda}$ (b) $\frac{1}{11\lambda}$ (c) $\frac{11}{10\lambda}$ (d) $\frac{1}{10\lambda}$

Shortcut

Undecayed = $\frac{N_0}{2^n}$

$t = nt_{1/2}$

Decayed	Undecayed	Time
$\frac{N_0}{2}$	$\frac{N_0}{2}$	$t_{1/2}$
$\frac{3N_0}{4}$	$\frac{N_0}{4} = \left(\frac{N_0}{2^2}\right)$	$2t_{1/2}$
$\frac{7N_0}{8}$	$\frac{N_0}{8} = \left(\frac{N_0}{2^3}\right)$	$3t_{1/2}$

No. of undecayed nuclei:

$N = \frac{N_0}{2^{t/t_{1/2}}}$

Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be:

(a) 1 : 4 (b) 5 : 4 (c) 1 : 16 (d) 4 : 1

	t_1	t_2
Undecayed	N_1	N_2

Time interval between 33% (1/3) & 67% (2/3) is $t_{1/2}$

$t_1 - t_2 = \frac{2.303}{\lambda} \log \frac{N_1}{N_2}$

Time interval between 20 % and 80% decay , or b/w 40 % and 85 % decay $(t_2 - t_1)$ is 2 $t_{1/2}$

The half-life of a radioactive substance is 30 min. The time (in minutes) taken between 40% decay & 85% decay of the same radioactive substance is:

(a) 15 (b) 60 (c) 45 (d) 30

Age of rock

X→Y (Y is stable)

Method 1

$\frac{N_x}{N_y} = \frac{m}{n}$ $N_0 = N_x + N_y = m+n$

$t = \frac{2.303}{\lambda} \log\left(\frac{m+n}{m}\right)$

Method 2

$\frac{N_0}{N} = 2^a = \frac{m+n}{m}$

Age= $at_{1/2}$

The half life of a radioactive isotope 'X' is 20 years. It decays to another element "Y" which is stable. The two elements 'X' and 'Y' were found to be in the ratio 1:7 in a sample of a given rock. The age of the rock is estimated to be

(a) 40 years (b) 60 years (c) 80 years (d) 100 years

Two deuterons undergo nuclear fusion to form a Helium nucleus. Energy released in this process is: (given binding energy per nucleon for deuteron-1.1 MeV and for helium=7.0 MeV)

(a) 30.2 MeV (b) 32.4 MeV (c) 23.6 MeV (d) 25.8 MeV

A radioactive nucleus (initial mass number A and atomic number Z emits 3 α - particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\frac{A-Z-8}{Z-4}$ (b) $\frac{A-Z-4}{Z-8}$ (c) $\frac{A-Z-12}{Z-4}$ (d) $\frac{A-Z-8}{Z-2}$

NUCLEAR FISSION

$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^1_0n$

NUCLEAR REACTOR

Multiplication Factor k=1 → critical

- Moderator : water, heavy water (D_2O), graphite and beryllium oxide.
- Control rods : Boron, cadmium
- Coolant : CO_2, water, nitrogen

NUCLEAR FUSION

$4\,^1_1H + 2\bar{e} \rightarrow ^4_2He + 2\nu + 6\gamma + 26.7$ MeV

Four hydrogen atoms combine to form an ^4_2He atom with the release of 26.7 MeV of energy

Achieved at very high temperature in order to overcome electrostatic repulsion